Symmetry, topology and coherence in condensed matter physics

Instructor: Leonid Levitov (levitov@mit.edu)

Lectures: Tue, Thur, 14:00-17:00

Office hours: by appointment

No textbook, reading material on stellar

Psets: 12 total, due Thuesday in lecture

Term paper: a list of term paper topics will be provided

Grade: psets 70%, term paper 30%

Course topics (tentative)

1. **Ordered matter**
Fields (phonons, spin waves).
Symmetry. Broken symmetry. Order parameter.
Geometry and topology of order parameter
Topological defects. Dislocations, domain walls, hedgehogs.

2. **Goldstone modes**
Continuous and discrete symmetries. Phonons, spin waves.
Phase transitions and symmetry breaking. Landau theory.
Examples: Magnetism. Superfluidity. Liquid crystals.
Anderson-Higgs mechanism
Loopholes in Goldstone theorem

3. **Hydrodynamics**
Conservation laws & symmetry
Collective modes: acoustic phonons, second sound, cosmic sound,
Spin waves in magnets, spin waves in Boltzmann gases

Course topics (tentative)

4. **Quantum mechanics of many-body systems**
Path integral in QM
Many-particle wavefunction
Second quantization
Bose-Einstein condensation
Field integral (bosons and fermions)

5. **Surprising orders (quantum)**
BCS pairing with nonzero angular momentum.
Symmetry of the order parameter.
Topological defects (hedgehogs, domain walls, boojums).
Homotopy groups. Classification

6. **Topology of Bloch bands.** Chern number and quantization of observables.

7. **Different types of symmetry breaking (SB).** Spontaneous SB and phase transitions. Superfluidity and magnetism.

Course topics (tentative)

8. **Anomalous SB.** Chiral anomaly and scaling anomaly in condensed matter systems.

9. **Interacting one-dimensional systems.** Luttinger liquids. Fractionalization of quantum numbers.

10. **Quantum Hall effects.** Edge transport. Topological order. Aharonov-Bohm effect and fractional statistics.

11. **Surprising orders (classical)**
Incommensurate crystals
phason modes
Frenkel-Kontorova
incommensurate/commensurate transition
Quasicrystals.

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Course topics

12. **Emergent symmetries Gauge symmetry**
Emergent gauge theories in CM.
Ising model, order-disorder duality.
Spin-ice systems, magnetic monopoles.
Dimers, tilings, dominos

13. **Topological defects and phase transitions**
vortices in the XY model, melting in 2D

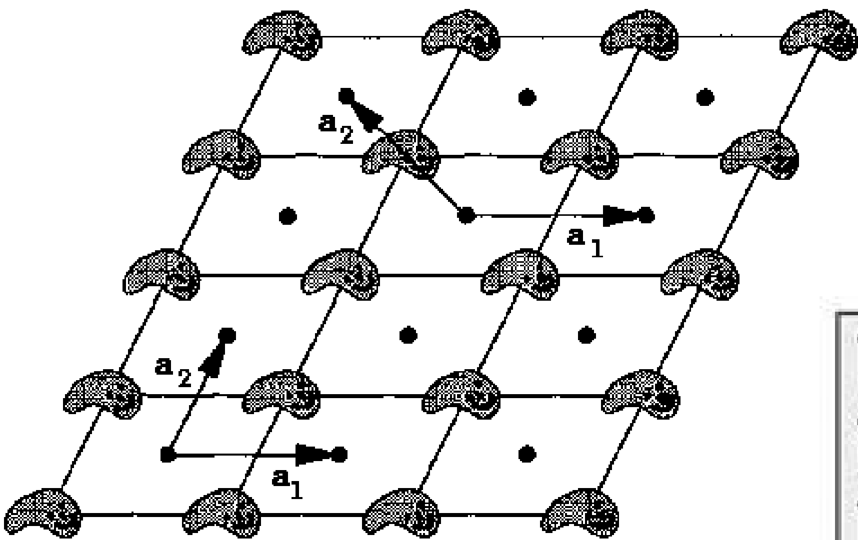
14. **Deformable crystals**
compressible lattices,
SL(2,Z) symmery, Phyllotaxis

15. **Hidden symmetries:**
space-time symmetries
Galilean transformation in solids
Lorentz symmetry in graphene.

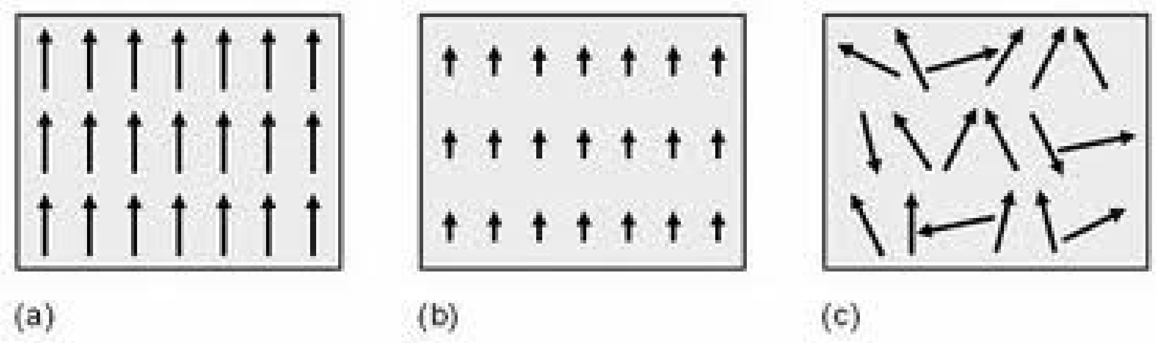
Course topics (tentative)

16. **Conformal symmetry**
Free fermion systems.
Interacting systems,
Quantum Ising model
Luttinger liquids,
Landau Fermi liquids

Ordered matter



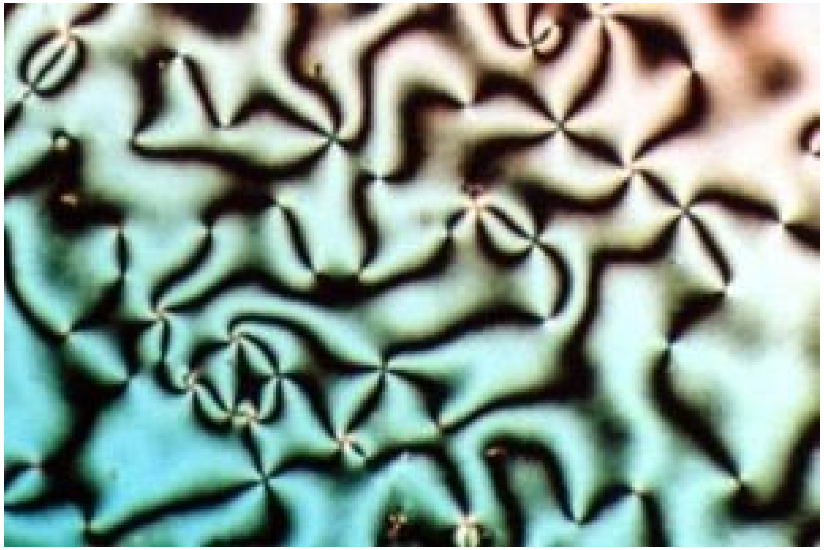
Spin order (magnets)



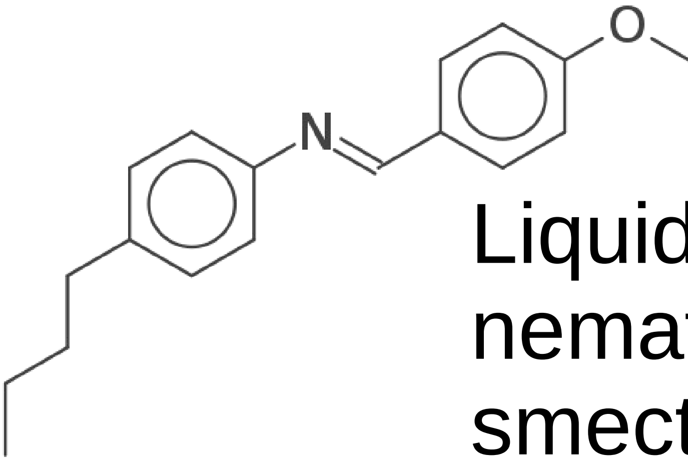
Crystalline order

Liquid crystals

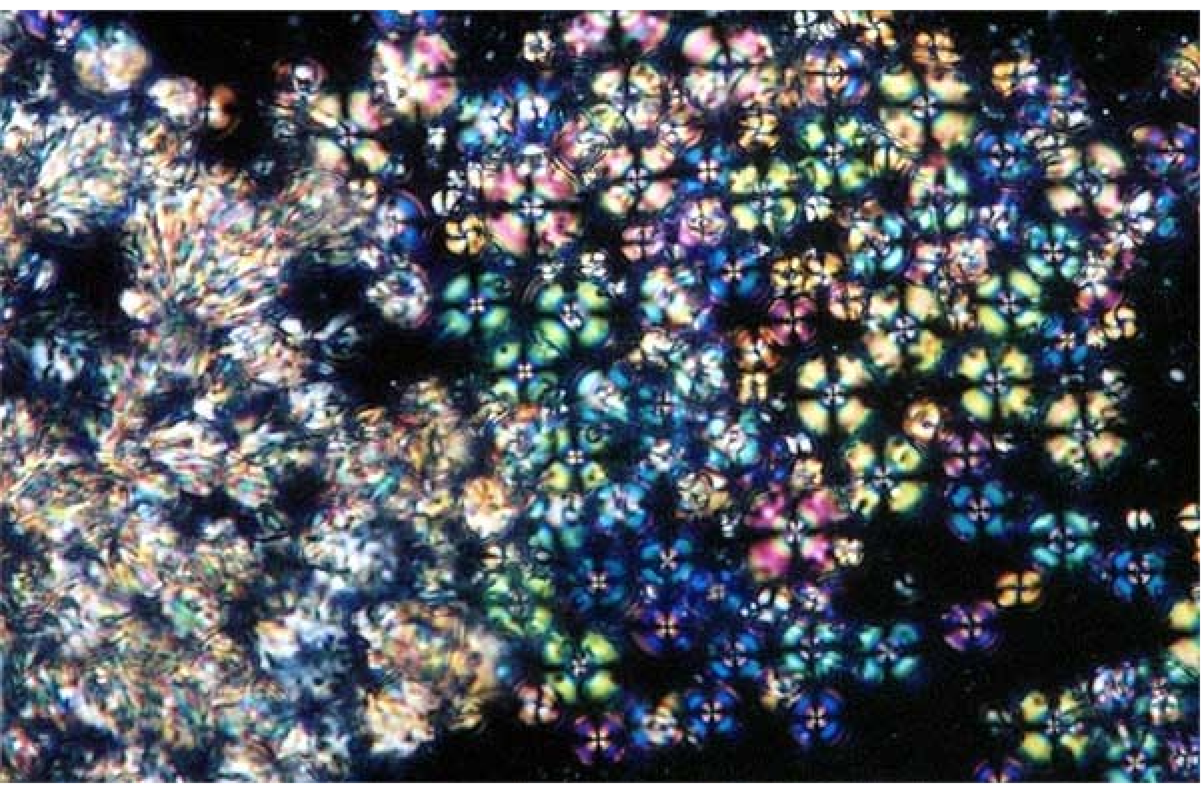
'peculiar double-melting liquid' discovered in 1888 (Friedrich Reinitzer) Explained and named 'flowing crystals' in 1904 (Otto Lehmann)

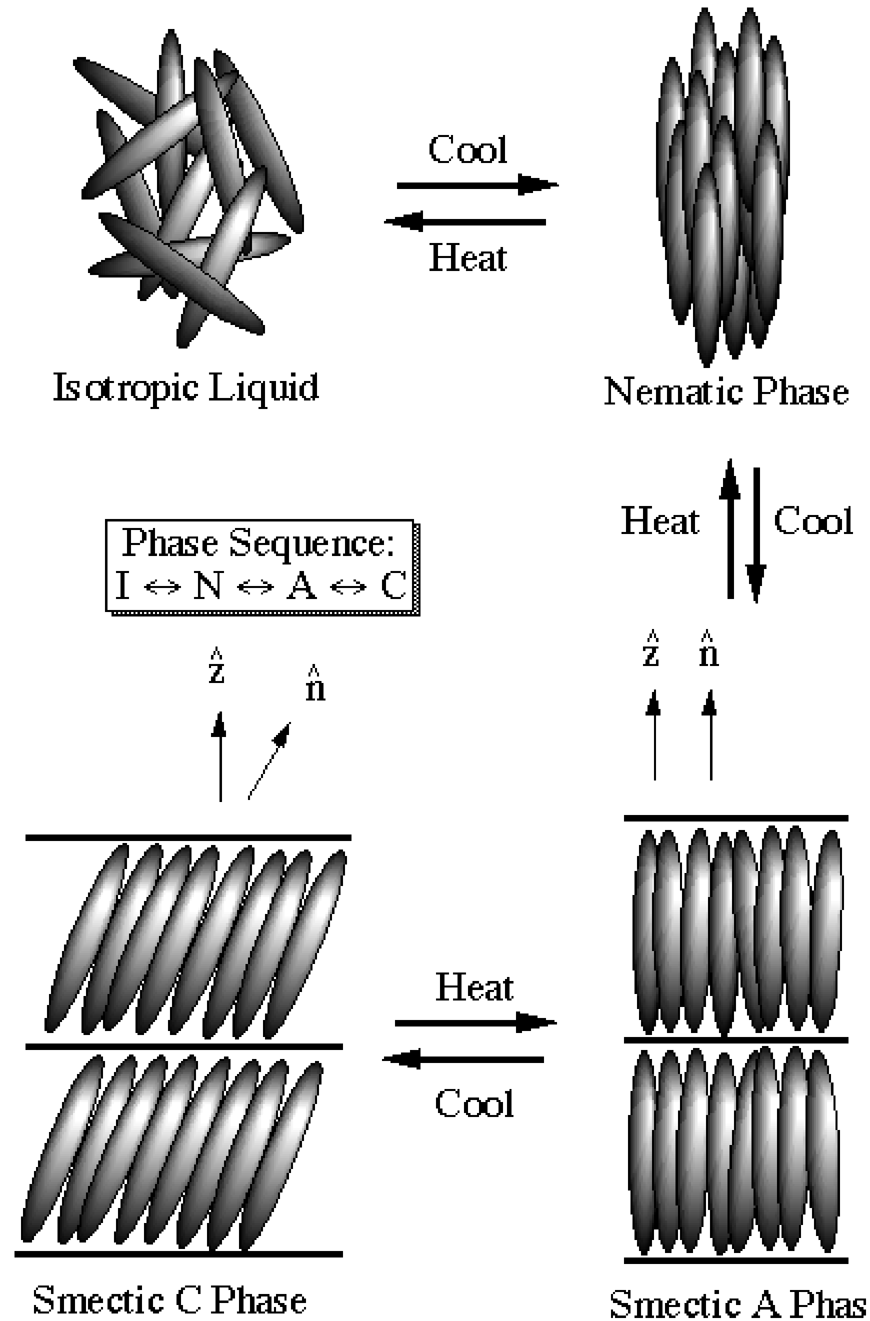


Ordered matter

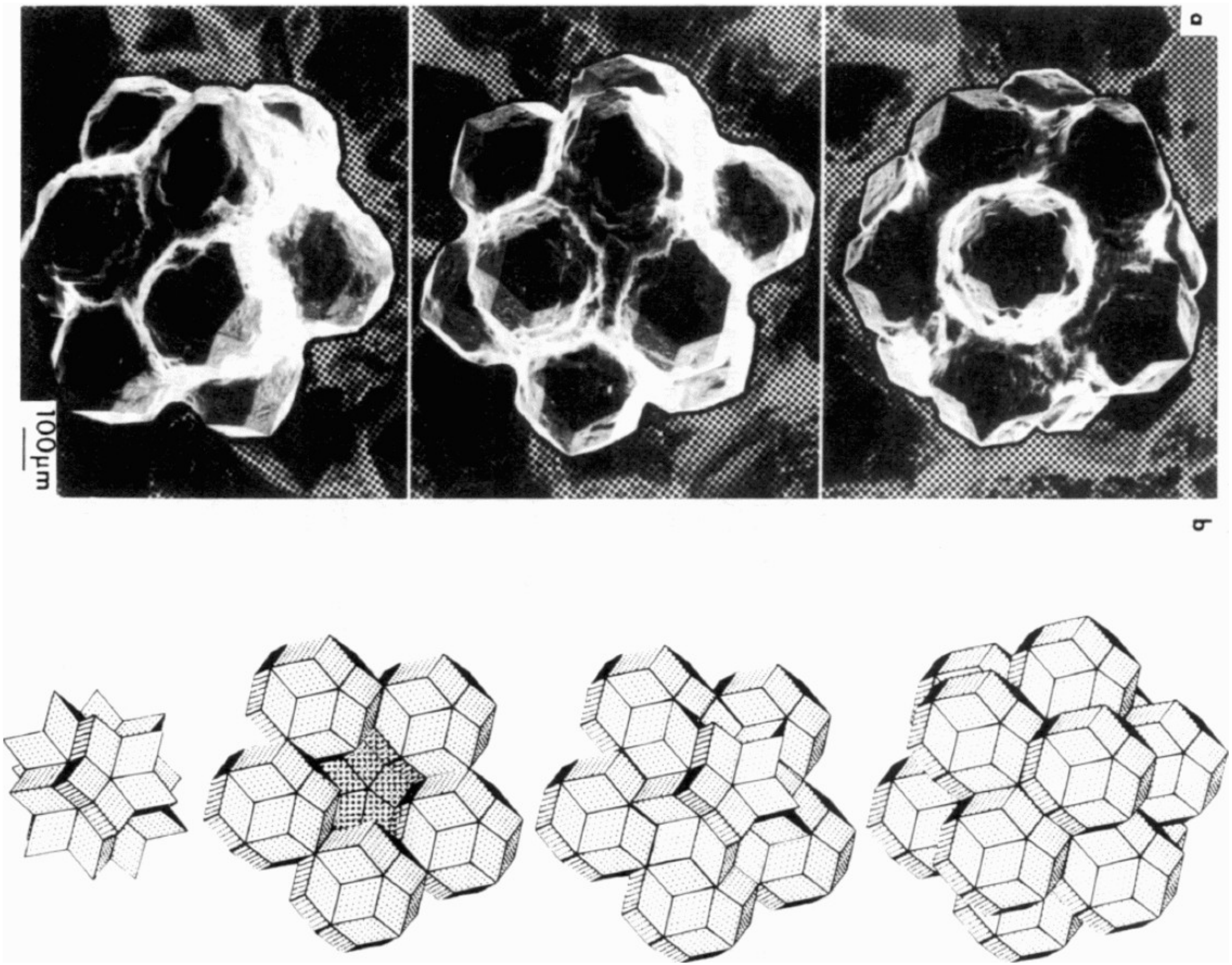


Liquid crystals
nematic
smectic
(various kinds)





Exotic orders: quasicrystals



Ordered matter: symmetry-based system

Order originates from broken symmetry (Landau & others)

Describes extremely well much of known ordered matter

Yet, a number of gaps in the symmetry-based system known: emergent symmetry, anomalous symmetry breaking, topological order

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One step at a time: first introduce system, then analyze gaps and exotic orders

Ordered matter: symmetry-based system

First, you must identify broken symmetry

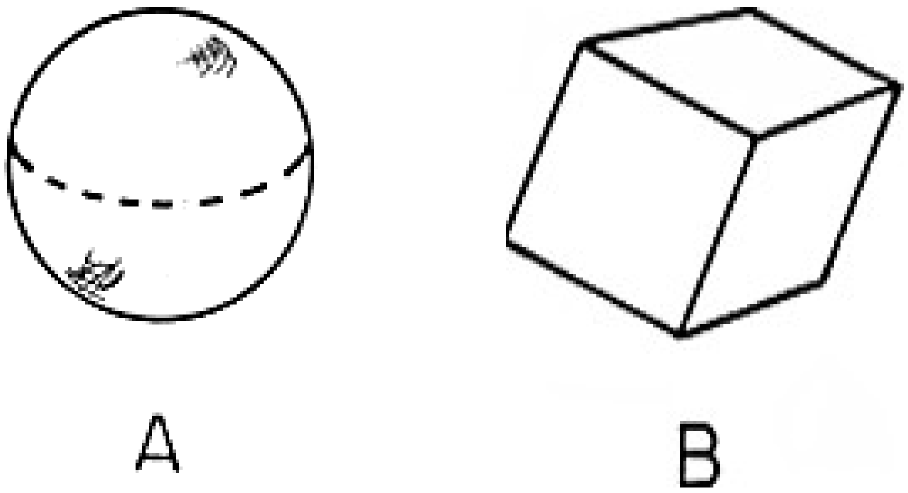
Second, you must define an order parameter

Third, you are told to examine elementary excitations

Fourth, you classify topological defects

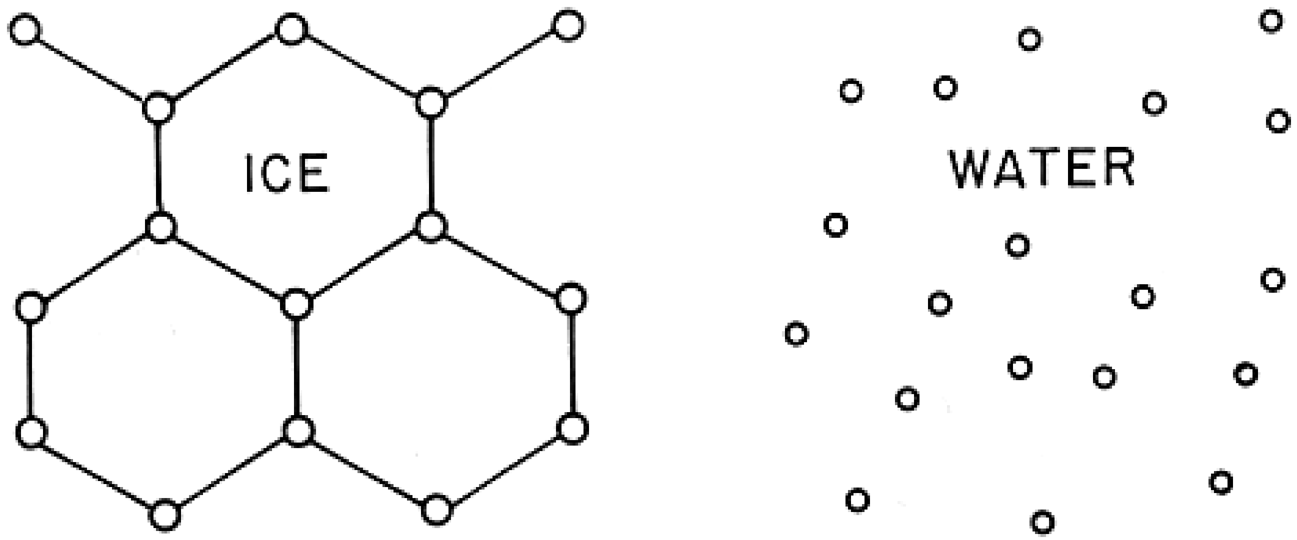
From: “Statistical Mechanics: Entropy, Order Parameters, and Complexity” by J. P. Sethna

Broken symmetry



Which is more symmetric?

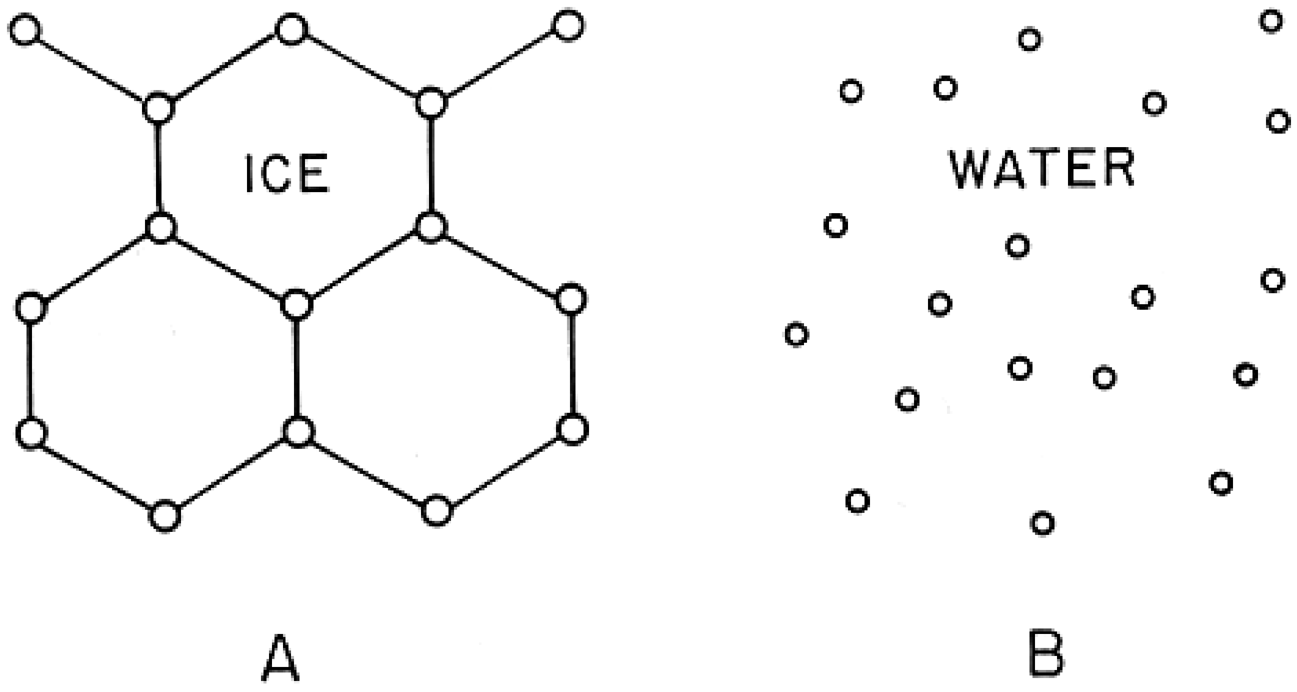
Broken symmetry



A B

Which is more symmetric?

Broken symmetry

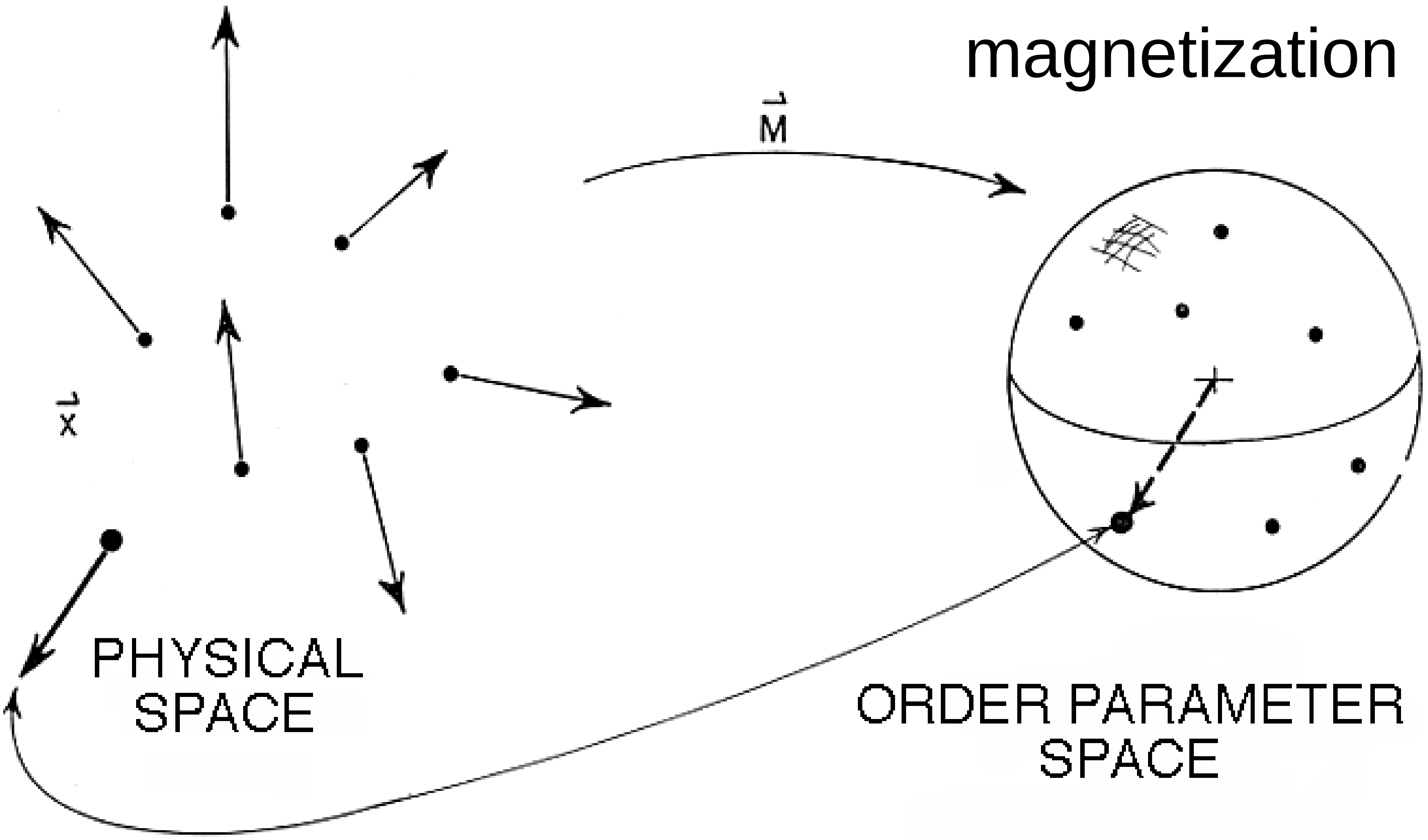


Which is more symmetric?

lower symmetry = higher order

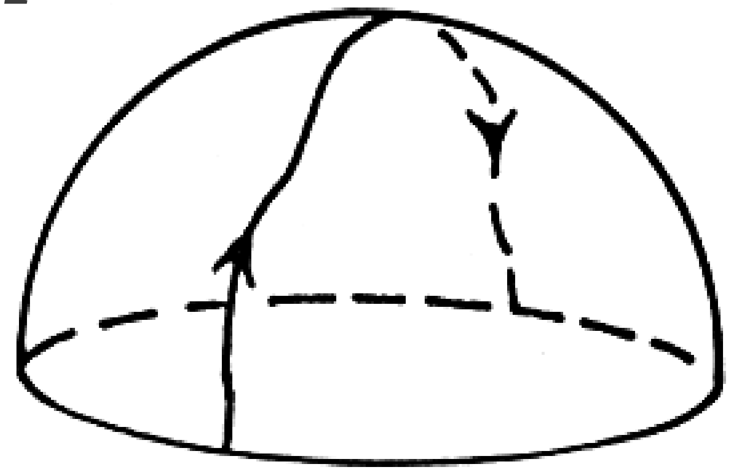
Order parameter

magnetization

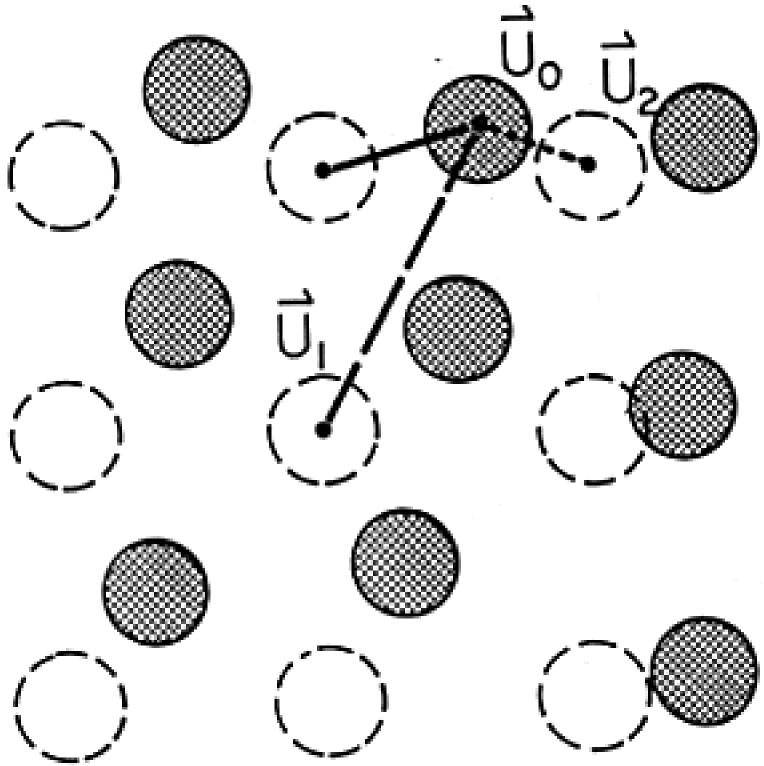


Order parameter for nematic LC: director (a headless vector)

$$\vec{n} \sim -\vec{n}$$

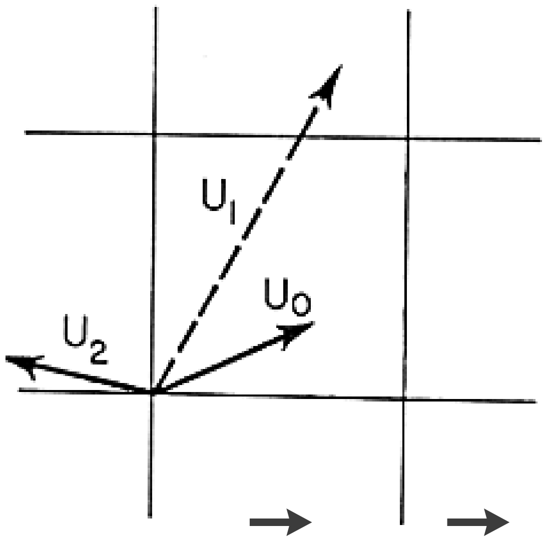


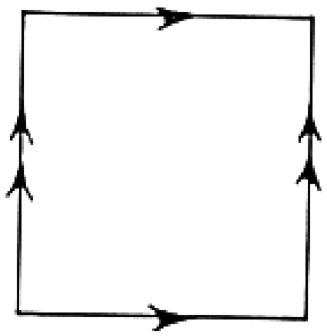
Crystal order parameter



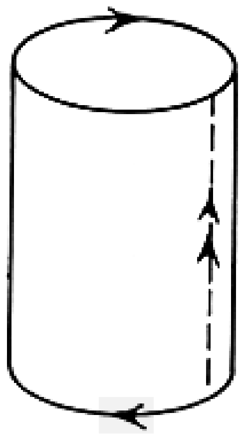
Displacement field

Order parameter space for a 2D crystal

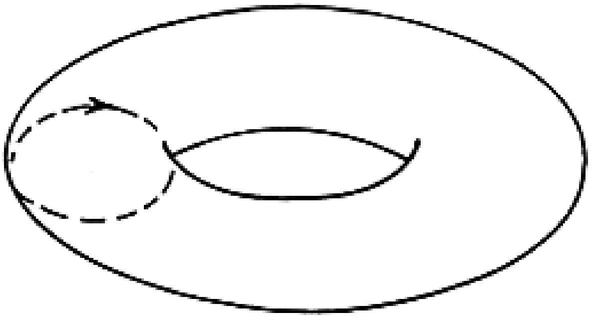




$$\vec{u} \sim \vec{u} + (ma, nb)$$



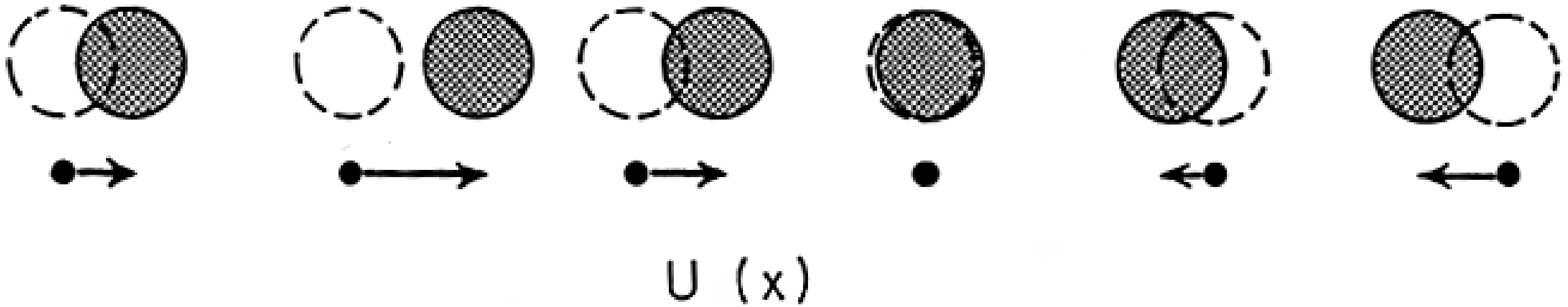
Torus



Elementary excitations

Order parameter as a field

Phonons



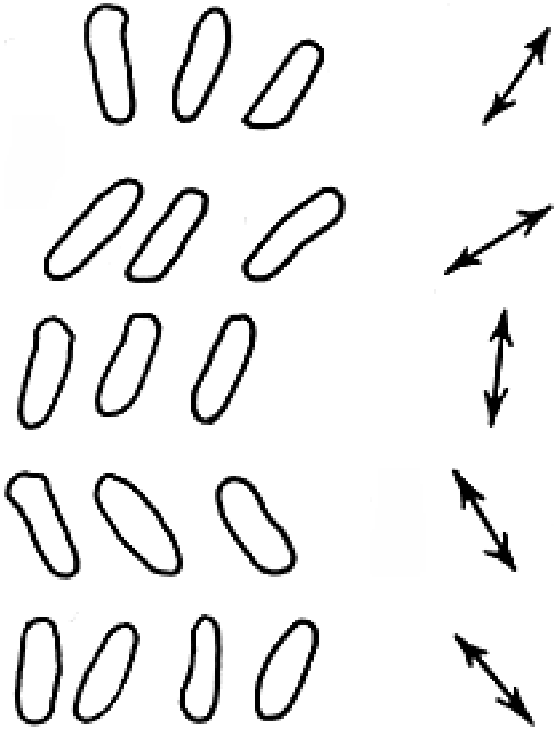
Spin waves

Elementary excitations in LC

Nematic rotational waves

Manifestation: liquid crystals opaque due to thermal fluctuations, undergo a transition in an external E field (1927, Vsevolod Frederiks)

Essential for LCD technology



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